

Answer ALL TWENTY THREE questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

- 1 Write 600 as a product of powers of its prime factors.
Show your working clearly.

$$\begin{aligned}600 &= 6 \times 100 \\ &= 2 \times 3 \times 10 \times 10 \\ &= 2 \times 3 \times 2 \times 5 \times 2 \times 5 \\ &= 2^3 \times 3 \times 5^2\end{aligned}$$

$$2^3 \times 3 \times 5^2$$

(Total for Question 1 is 3 marks)



2 Show that $2\frac{4}{7} \div 1\frac{1}{8} = 2\frac{2}{7}$

$$2\frac{4}{7} = \frac{18}{7}$$

$$1\frac{1}{8} = \frac{9}{8}$$

$$2\frac{4}{7} \div 1\frac{1}{8}$$

$$= \frac{18}{7} \div \frac{9}{8}$$

$$= \frac{18}{7} \times \frac{8}{9}$$

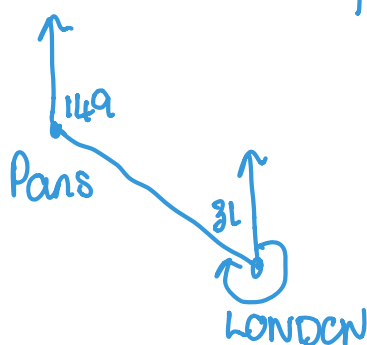
$$= \frac{16}{7}$$

$$\frac{16}{7} = 2\frac{2}{7} \text{ as required}$$

(Total for Question 2 is 3 marks)

3 The bearing of Paris from London is 149°

Work out the bearing of London from Paris.



$$180 - 149 = 31$$

$$360 - 31 = 329$$

329

(Total for Question 3 is 2 marks)



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4 $\mathcal{E} = \{\text{letters of the alphabet}\}$

$B = \{\text{b, r, a, z, i, l}\}$

$I = \{\text{i, r, e, l, a, n, d}\}$

(a) List the members of the set

(i) $B \cup I$

b r a z i l e n d

(ii) $B \cap I'$

b z

(2)

$K = \{\text{k, e, n, y, a}\}$

$B = \text{brazil}$

Cody writes down the statement $B \cap K = \emptyset$

Cody's statement is wrong.

(b) Explain why.

It is not an empty set. The letter 'a' is in both B and K

(1)

(Total for Question 4 is 3 marks)



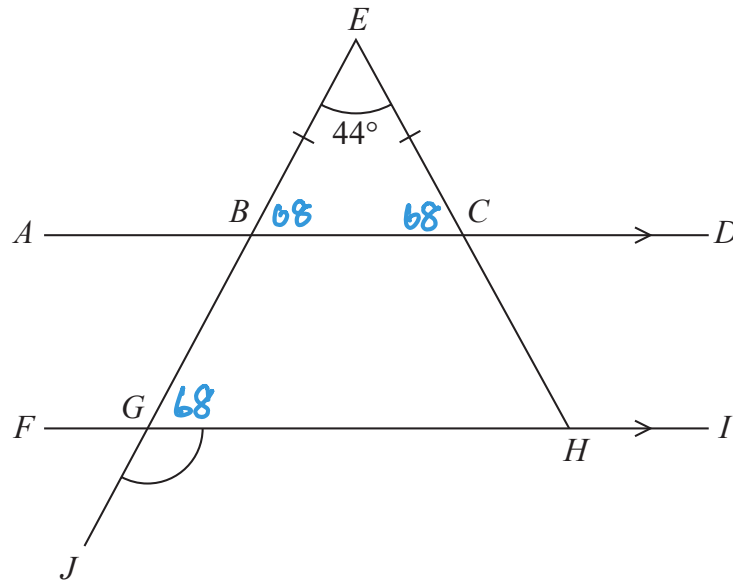


Diagram **NOT**
accurately drawn

$ABCD$ and $FGHI$ are parallel straight lines.
 $EBGJ$ and ECH are straight lines.

$$BE = CE$$

$$\text{Angle } BEC = 44^\circ$$

Work out the size of angle JGH .

Give a reason for each stage of your working.

$$180 - 44 = 136 \quad 136 \div 2 = 68$$

2 angles in an isosceles are equal

$$\angle BGC = 68 \quad \text{its a corresponding angle (which are equal)}$$

$$\angle JGH = 180 - 68 = 112 \quad \text{angles on a straight line} = 180$$

112

(Total for Question 5 is 5 marks)



6 Mariana sells bags of bird food.

The bags that Mariana sold last week each contained 12 kg of seeds.

The bags that she is going to sell next week will each contain a mixture of nuts and seeds where for each bag

$$\text{weight of nuts : weight of seeds} = 4 : 5$$

The total weight of the nuts and the seeds in each bag will be 19.35 kg

The weight of seeds in each bag that Mariana sells next week will be less than the weight of seeds in each bag that Mariana sold last week.

Work out this decrease as a percentage of the weight of seeds in each bag that Mariana sold last week.

Give your answer correct to one decimal place.

last week

$$12 \text{ kg} = \text{seeds}$$

next week.

$$\text{nuts : seeds}$$

$$4 : 5$$

$$\underbrace{\hspace{10em}}$$

$$19350 \div 9 = 2150$$

$$8600 : 10750$$

$$\text{Decrease} = 12 - 10.75 = 1.25 \text{ kg}$$

$$\% = \frac{1.25}{12} \times 100$$

$$= 10.4166..$$

↑

(1dp)

10.4

%

(Total for Question 6 is 4 marks)



7 Here is a right-angled triangle.

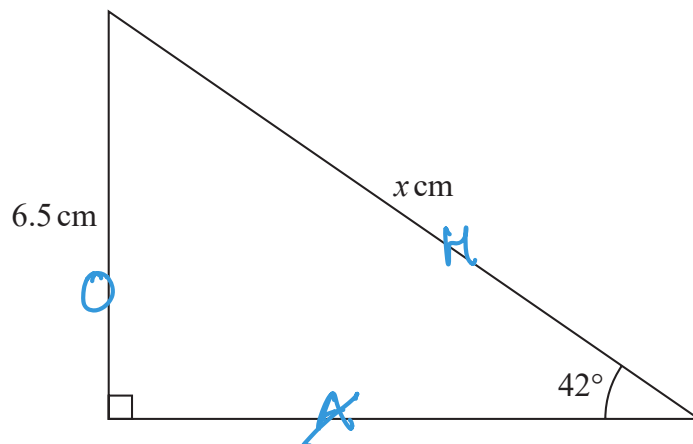


Diagram NOT accurately drawn

Work out the value of x .
Give your answer correct to one decimal place.

$$\sin 42 = \frac{6.5}{x}$$

$$\begin{aligned} x &= \frac{6.5}{\sin 42} \\ &= 9.714\dots \\ &\quad \uparrow \\ &\quad (1\text{dp}) \end{aligned}$$

$$x = 9.7$$

(Total for Question 7 is 3 marks)

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8 Solve the simultaneous equations

$$\begin{aligned} \textcircled{1} \quad 5a + 2c &= 10 \quad \times 2 \\ \textcircled{2} \quad 2a - 4c &= 7 \end{aligned}$$

Show clear algebraic working.

$$\begin{array}{r} 10a + 4c = 20 \quad \textcircled{3} \\ 2a - 4c = 7 \quad \textcircled{2} \\ \hline + \quad 12a \quad \times = 27 \\ a = \frac{27}{12} = 2.25 \end{array}$$

sub in $\textcircled{2}$

$$2 \times 2.25 - 4c = 7$$

$$4.5 - 7 = 4c$$

$$c = \frac{-2.5}{4}$$

$$= -0.625$$

$$a = 2.25$$

$$c = -0.625$$

(Total for Question 8 is 3 marks)

9 (i) Factorise $x^2 + 2x - 24$

$$1, 24 \quad 2, 12 \quad 3, 8 \quad 4, 6$$

$$6 - 4$$

$$(x + 6)(x - 4)$$

$$\frac{(x + 6)(x - 4)}{(2)}$$

(ii) Hence solve $x^2 + 2x - 24 = 0$

$$(x + 6)(x - 4) = 0$$

$$\begin{array}{cc} \downarrow & \downarrow \\ -6 & 4 \end{array}$$

$$x = 4, x = -6$$

(1)

(Total for Question 9 is 3 marks)



10 Here is a triangular prism.

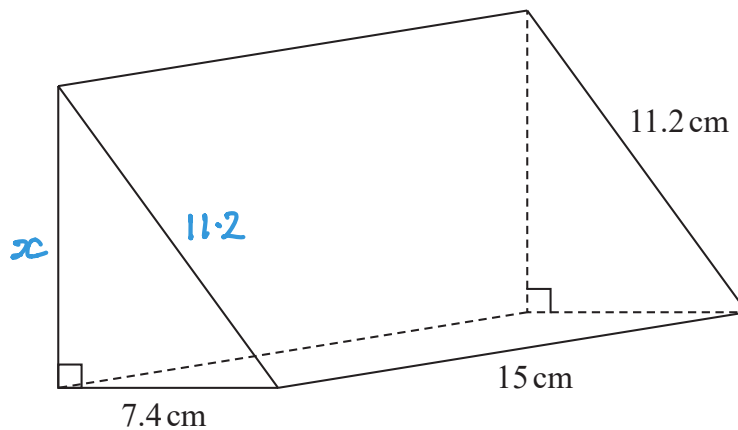


Diagram **NOT** accurately drawn

Work out the volume of the prism.
Give your answer correct to 3 significant figures.

$$x = \sqrt{11.2^2 - 7.4^2}$$
$$= 8.40713\dots$$

$$\text{Volume} = \frac{1}{2} \times 7.4 \times 8.40\dots \times 15$$
$$= 466.596\dots$$

↑
(2sf)

467 cm³

(Total for Question 10 is 5 marks)

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11 Chengbo sold a house for 180 000 yuan.

The amount for which he sold the house is 24% more than the amount he paid for the house.

- (a) Work out how much Chengbo paid for the house.
Give your answer correct to 3 significant figures.

$$\begin{aligned}
 124\% &= 180\,000 \\
 \div 124 &\downarrow \\
 1\% &= 1451.612\dots \\
 \times 100 &\downarrow \\
 100\% &= 145,161.29 \\
 &\uparrow \\
 &(3\text{ sf})
 \end{aligned}$$

145,000 yuan
(3)

Zhi bought a house on 1st January 2017
When she bought the house, its value was 120 000 yuan.

The value of the house increased by 1.8% per year.

- (b) Work out the value of Zhi's house on 1st January 2020
Give your answer correct to 3 significant figures.

$$\begin{aligned}
 &2017 \quad \curvearrowright \quad 2018 \quad \curvearrowright \quad 2019 \quad \curvearrowright \quad 2020 \quad \text{3 years} \\
 &120\,000 \\
 &120\,000 \times 1.018^3 \\
 &= 126\,597.3398 \\
 &\quad \uparrow \\
 &\quad (3\text{ sf})
 \end{aligned}$$

127 000 yuan
(3)

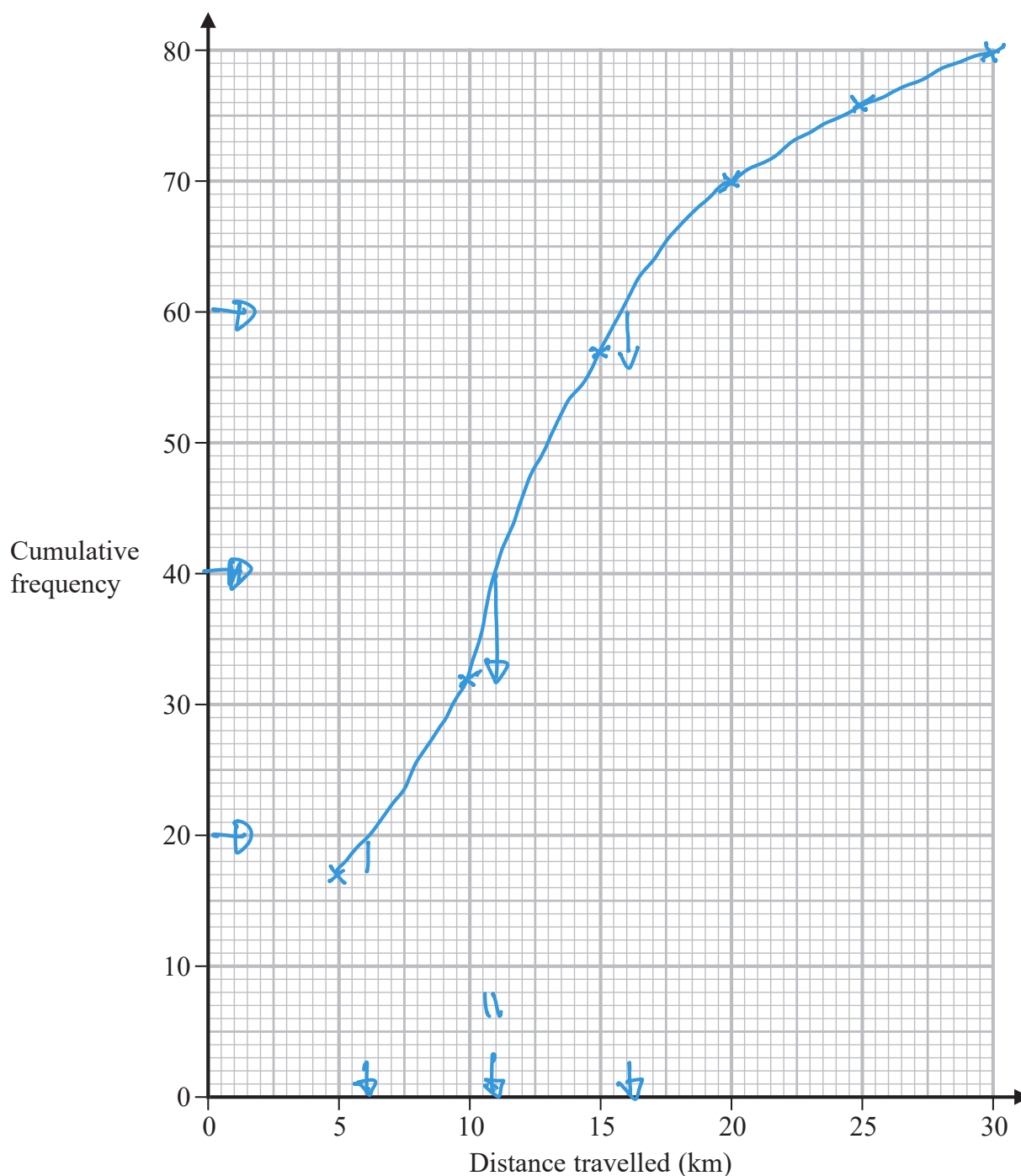
(Total for Question 11 is 6 marks)



- 12 The cumulative frequency table gives information about the distance, in kilometres, that each of 80 workers travel from home to work at Office *A*.

Distance travelled (d km)	Cumulative frequency
$0 < d \leq 5$	17
$0 < d \leq 10$	32
$0 < d \leq 15$	57
$0 < d \leq 20$	70
$0 < d \leq 25$	76
$0 < d \leq 30$	80

- (a) On the grid below, draw a cumulative frequency graph for the information in the table.



(2)



(b) Use your graph to find an estimate for the median distance travelled.

(range accepted
10.5 to 12)

..... 11 km
(1)

(c) Use your graph to find an estimate for the interquartile range of the distances travelled.

$$LQ = 6$$

$$16 - 6$$

(range accepted
8.5 to 11.5)

$$UQ = 16$$

..... 10 km
(2)

For Office *B*, the median distance workers travel from home to work is 15 km and the interquartile range is 5 km.

$$A = 10$$

$$A = 11$$

(d) Use the information above to compare the distances that workers at Office *A* and workers at Office *B* travel from home to work.

Write down **two** comparisons.

1 The median for office B is higher so on average they travel further.

2 The IQR for office B is lower so the distances are less spread out.

(2)

(Total for Question 12 is 7 marks)



Emilie is going to take part in a third race.

If she wins both of the first two races, the probability that she will win the third race is 0.6 $L=0.4$

If she wins exactly one of the first two races, the probability that she will win the third race is 0.3

(c) Work out the probability that Emilie will win exactly two of the three races.

$$P(W, W, L) = 0.7 \times 0.4 \times 0.4 = 0.112$$

$$P(\text{Wins one of two}, W) = 0.54 \times 0.3 = 0.162$$

$$0.112 + 0.162$$

0.274

(3)

(Total for Question 13 is 8 marks)



14 Simplify fully $\left(\frac{9x^4}{16y^{10}}\right)^{-\frac{1}{2}}$

$$\Rightarrow \left(\frac{16y^{10}}{9x^4}\right)^{\frac{1}{2}}$$

$$= \frac{\sqrt{16} y^{10 \times 0.5}}{\sqrt{9} x^{4 \times 0.5}} = \frac{4y^5}{3x^2}$$

$$\frac{4y^5}{3x^2}$$

(Total for Question 14 is 3 marks)

15 (a) Complete the table of values for $y = \frac{1}{x}(x^2 + 4)$

x	0.25	0.5	1	2	4	8
y	16.25	8.5	5	4	5	8.5

$$\frac{1}{0.5}(0.5^2 + 4)$$

$$\frac{1}{1}(1^2 + 4)$$

$$\frac{1}{2}(2^2 + 4)$$

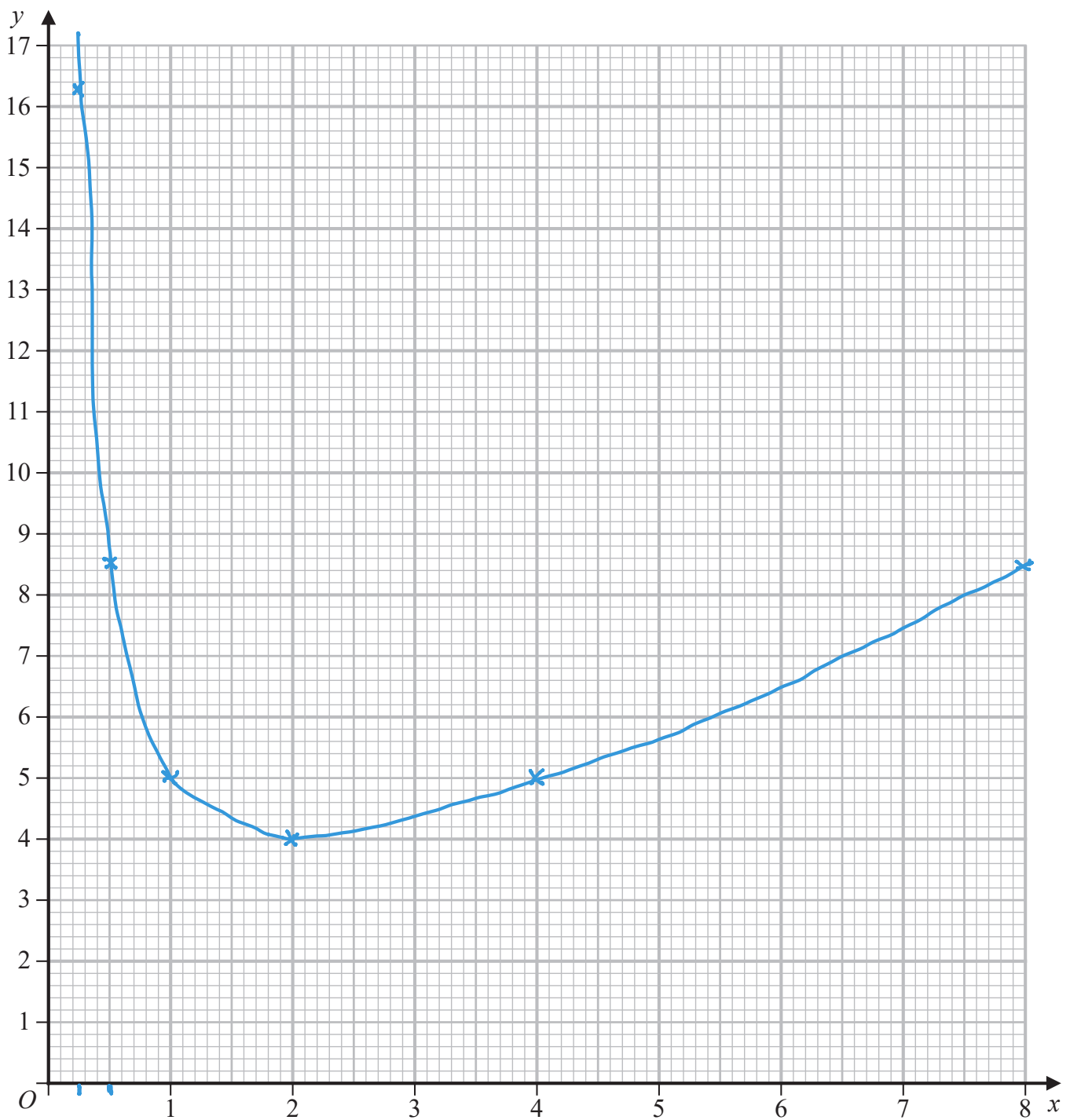
$$= \frac{1}{2} \times 8$$

$$\frac{1}{4}(4^2 + 4)$$

$$= \frac{1}{4} \times 20$$



(b) On the grid, draw the graph of $y = \frac{1}{x}(x^2 + 4)$ for $0.25 \leq x \leq 8$



(2)

(Total for Question 15 is 4 marks)

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16 A is inversely proportional to the square of r

$$A = 5 \text{ when } r = 0.3$$

(a) Find a formula for A in terms of r

$$A \propto \frac{1}{r^2}$$

$$A = \frac{k}{r^2}$$

$$5 = \frac{k}{0.3^2}$$

$$k = 5 \times 0.3^2 \\ = 0.45$$

$$\therefore A = \frac{0.45}{r^2}$$

$$A = \frac{0.45}{r^2} \dots\dots\dots (3)$$

(b) Find the value of A when $r = 7.5A$

$$A = \frac{0.45}{(7.5A)^2}$$

$$= \frac{0.45}{56.25A^2}$$

$$A^3 = \frac{0.45}{56.25}$$

$$A = \sqrt[3]{\frac{0.45}{56.25}} = 0.2$$

$$A = 0.2 \dots\dots\dots (3)$$

(Total for Question 16 is 6 marks)



17 The straight line **L** passes through the points (4, -1) and (6, 4)

The straight line **M** is perpendicular to **L** and intersects the y-axis at the point (0, 8)

Find the coordinates of the point where **M** intersects the x-axis.

$$\text{gradient of } h \Rightarrow \frac{4 - (-1)}{6 - 4}$$

$$= \frac{5}{2}$$

$$\text{perp. line} = -\frac{2}{5} \quad (0, 8) \text{ \& intersect}$$

$$\therefore m \quad y = -\frac{2}{5}x + 8$$

crosses x axis when $y = 0$

$$\frac{2}{5}x = 8$$

$$x = \frac{8 \times 5}{2} = 20$$

(.....20....., 0.....)

(Total for Question 17 is 4 marks)



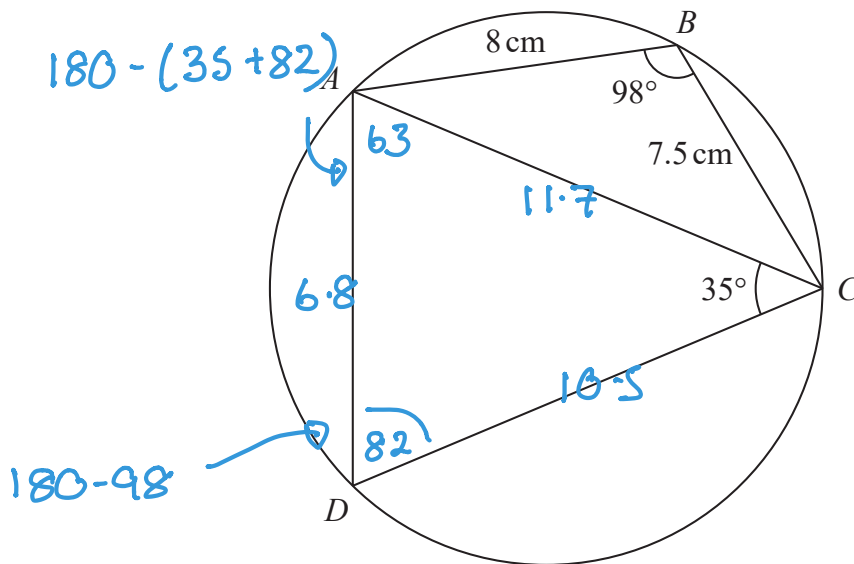


Diagram **NOT**
accurately drawn

$ABCD$ is a quadrilateral where A , B , C and D are points on a circle.

$$AB = 8 \text{ cm}$$

$$BC = 7.5 \text{ cm}$$

$$\text{Angle } ABC = 98^\circ$$

$$\text{Angle } ACD = 35^\circ$$

Work out the perimeter of quadrilateral $ABCD$.

Give your answer correct to one decimal place.

$$\begin{aligned} AC^2 &= 8^2 + 7.5^2 - 2 \times 8 \times 7.5 \times \cos 98 \\ &= 136.95 \dots \end{aligned}$$

$$AC = 11.70259 \dots$$

$$\frac{11.7}{\sin 82} = \frac{AD}{\sin 35} = \frac{DC}{\sin 63}$$

$$\begin{aligned} AD &= \frac{11.7}{\sin 82} \times \sin 35 \\ &= 6.7767 \dots \end{aligned}$$

$$\begin{aligned} DC &= \frac{11.7}{\sin 82} \times \sin 63 \\ &= 10.5272 \dots \end{aligned}$$



$$\begin{aligned} \text{Perimeter} &= 6.8 + 10.5 + 8 + 7.5 \\ &= 32.8 \quad (1 \text{ dp}) \end{aligned}$$

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..... 32.8 cm

(Total for Question 18 is 6 marks)



19 Solve the simultaneous equations

$$\begin{aligned}y &= 3 - 2x \\ x^2 + y^2 &= 18\end{aligned}$$

Show clear algebraic working.

$$y^2 = (3 - 2x)(3 - 2x) = 9 - 12x + 4x^2$$

$$x^2 + 9 - 12x + 4x^2 - 18 = 0$$

$$5x^2 - 12x + 9 = 0$$

$$(5x + 3)(x - 3) = 0$$

$$x = -\frac{3}{5} \quad \text{and} \quad x = 3$$

$$y = 3 - 2\left(-\frac{3}{5}\right)$$

$$= 4.2$$

$$\begin{aligned}y &= 3 - 2 \times 3 \\ &= -3\end{aligned}$$

$$\begin{array}{ll}x = -0.6 & x = 3 \\ y = 4.2 & y = -3\end{array}$$

(Total for Question 19 is 5 marks)

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20 Mathematically similar wooden blocks are made in a workshop.

There are small blocks and there are large blocks.

The volume of each small block is 300 cm^3

Given that

the surface area of each small block : the surface area of each large block = $25 : 36$

work out the volume of each large block.

$$\text{area scale factor} = 1.44$$

$$\begin{aligned} \text{length SF} &= \sqrt{1.44} \\ &= 1.2 \end{aligned}$$

$$\begin{aligned} \text{volume SF} &= 1.2^3 \\ &= 1.728 \end{aligned}$$

$$\begin{aligned} V_{\text{LARGE}} &= 300 \times 1.728 \\ &= 518.4 \end{aligned}$$

..... 518.4 cm^3

(Total for Question 20 is 3 marks)



- 21 The point A is the only stationary point on the curve with equation $y = kx^2 + \frac{16}{x}$ where k is a constant.

Given that the coordinates of A are $\left(\frac{2}{3}, a\right)$

find the value of a .

Show your working clearly.

$$y = kx^2 + 16x^{-1}$$

$$\frac{dy}{dx} = 2kx - 16x^{-2}$$

$$A = \left(\frac{2}{3}, a\right)$$

$x \quad y$

$$2kx - 16x^{-2} = 0$$

$$2kx = \frac{16}{x^2}$$

$$k = \frac{16 \cdot 8}{2x^3}$$

$$= \frac{8}{x^3}$$

$$k = \frac{8}{\left(\frac{2}{3}\right)^3} = 27$$

$$\therefore y = 27x^2 + \frac{16}{x}$$

$$x = \frac{2}{3} = 27 \times \left(\frac{2}{3}\right)^2 + \frac{16}{2/3}$$

$$= 36$$

$$a = 36$$

(Total for Question 21 is 5 marks)



- 22 The curve **S** has equation $y = f(x)$ where $f(x) = x^2$
The curve **T** has equation $y = g(x)$ where $g(x) = 2x^2 - 12x + 13$

By writing $g(x)$ in the form $a(x - b)^2 - c$, where a , b and c are constants, describe fully a series of transformations that map the curve **S** onto the curve **T**.

$$\begin{aligned}g(x) &= 2(x^2 - 6x) + 13 \\&= 2[(x - 3)^2 - 9] + 13 \\&= 2(x - 3)^2 - 18 + 13 \\&= 2(x - 3)^2 - 5\end{aligned}$$

Stretch in y using a scale factor of 2
followed by a translation $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$

(Total for Question 22 is 4 marks)



23 Pippa has a box containing N pens.

There are only black pens and red pens in the box.

The number of black pens in the box is 3 more than the number of red pens.

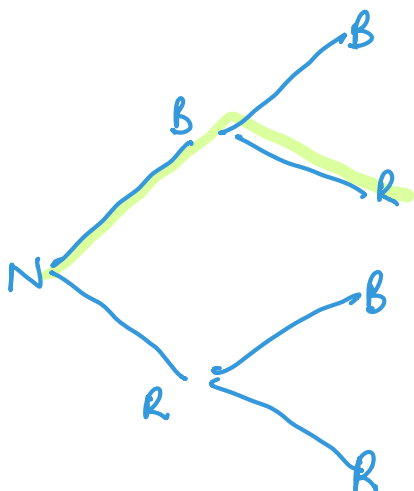
$$\begin{aligned} \text{Red} &= R \\ \text{Black} &= R + 3 \end{aligned}$$

Pippa is going to take at random 2 pens from the box.

The probability that she will take a black pen **followed** by a red pen is $\frac{9}{35}$

Find the possible values of N .

Show clear algebraic working.



$$P(B, R) = \frac{9}{35}$$

$$\Rightarrow \frac{R+3}{N} \times \frac{R}{N-1}$$

$$\begin{aligned} \text{But: } N &= R + R + 3 \\ &= 2R + 3 \end{aligned}$$

$$\text{so } \frac{R+3}{2R+3} \times \frac{R}{2R+3-1} = \frac{9}{35}$$

$$\frac{R+3}{2R+3} \times \frac{R}{2R+2} = \frac{9}{35}$$

$$35R(R+3) = 9(2R+3)(2R+2)$$

$$\begin{aligned} 35R^2 + 105R &= 9(4R^2 + 10R + 6) \\ &= 36R^2 + 90R + 54 \end{aligned}$$

$$0 = 36R^2 - 35R^2 + 90R - 105R + 54$$



$$R^2 - 15R + 54 = 0$$

$a \times b$

$$(R - a)(R - b) = 0$$

$$\downarrow \\ R = 9$$

$$\downarrow \\ R = 6$$

$$N = 2R + 3$$

$$N = 21$$

$$N = 15$$

..... 21, 15

(Total for Question 23 is 5 marks)

TOTAL FOR PAPER IS 100 MARKS

